Automated Mode Tracking Strategy

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Introduction

A NORMAL mode analysis of a discrete system results in a sequence of frequencies and their corresponding mode shapes. These frequencies and mode shapes are normally put in a numbered order, sequentially increasing with increasing modal frequency. This ad hoc modal numbering convention enables automated iterative model-updating procedures to keep track of the modes. A more precise way of modal identification would be to look at the graphic display of each mode shape and give it a name according to its physical features. This approach has many advantages, but it requires a highly experienced vibration engineer to do it right; and there is not as yet an artificial intelligence which can do it.

A frequently encountered problem in any iterative optimization procedure is how to keep track of correct modal identification when design changes introduced after each iteration give rise to a new set of natural frequencies and mode shapes in which two or more of frequencies switch their positions in the ordered sequence. When this occur unrecognized, the modal properties of the switched modes appear to be discontinuous. Any further improvements on these modes become meaningless, since they are now being targeted to the wrong target modes. Such a problem is commonly known as the mode crossing condition. There are, so far, very few works which have addressed this problem.

A related mode tracking problem is the mode coupling condition which is caused by the rapidly changing behavior of crossing modes during the crossing interval. A solution to this problem is also discussed.

The present work[1] focuses on the development of a method which solves the problem, in a practical manner, with a minimum increase in computer resources. Furthermore, the method produces a modal coordinate transformation matrix which is useful in transforming any modal related matrices into a correct order.

Solution Techniques

To solve nonlinear problem, an iterative procedure, is assumed which modifies design parameters based on system modal performance requirements. Let the baseline eigenmodes be ordered based on the solution of the initial modal analysis and assume they are numbered from 1, for the lowest frequency mode, through s. If I is the set of ordered numbers $i = 1, 2, \ldots, s$ corresponding to the baseline modal numbers, then J represents a subset of I which contains t modal numbers, $j = j_1, j_2, \ldots, j_t$, of the targeted baseline eigenmodes.

Initial Iteration

The initial targeted modes are partitioned out of the extracted eigenmodes by

$$[\Phi^0]_J = [\Phi^0]_I [P^0]_{JI}^T \tag{1}$$

where $[\Phi]_I$ is the extracted modal matrix, $[\Phi]_J$ the targeted modal matrix, and $[P]_{JI}$ the corresponding modal coordinate transformation matrix containing 0s and 1s. The initial $[P]_{JI}$, i.e., $[P^0]_{JI}$, is simply formed based on the set of modal numbers corresponding to the baseline eigenmodes chosen by the user. This operation allows for arranging the targeted modes in any order. For example, if the baseline eigenmodes contain three modes, i.e., $s = 3$, and, among them, only two modes are targeted modes, with the correspondence order of $j_1 = 3$ and $j_2 = 1$, the corresponding partition operator is

$$[P^0]_{JI} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

After the partition operation, $[\Phi]_J$ is stored in a database which allows the stored information to be passed on to the next iteration.

Subsequent Iterations

The same operation is carried out in all of the subsequent iterations. For the kth iteration, where $k = 1, 2, \ldots,$ the following steps are performed:

1) Fetch $[\Phi^{k-1}]_J$ from the database.

2) Determine the modal assurance coefficient (MAC) matrix, relating the mode shapes of the two successive iterations.

$$[\text{MAC}]_{JI} = \left[\hat{\Phi}^{k-1}\right]_J^T \left[\hat{\Phi}^k\right]_I \tag{2}$$

where $[\hat{\Phi}]$ represents the normalized $[\Phi]$ with each column being unitary.

3) Determine the Boolean operator $[P^k]_{JI}$. First, the MAC matrix is normalized to have the maximum value in each row be 1. The normalized MAC matrix is then filtered by replacing terms smaller than 1, with 0s and the resulting matrix is the Boolean operator $[P^k]_{JI}$.

4) Partition targeted modes. A new set of modal vectors can now be partitioned and rearranged in the correct order with respect to the baseline eigenmodes by

$$[\Phi^k]_J = [\Phi^k][P^k]_{JI}^T \tag{3}$$

5) Store $[\Phi^k]_J$ into the database.

Dealing with Coupled Modes

The described approach usually can keep track of modes which cross each other, as long as the iterations give eigenvalue solutions which either precede or follow the crossing. For solutions which land in the middle of a crossing, however, the fast-changing crossing mode shapes often become temporarily unrecognizable. In a mode crossing (of a fully coupled system) the two modes do not cross in frequency but, instead, rapidly interchange mode shapes with each other. The two modes are said to be coupled together during this interval and can be extremely difficult, if not impossible, to track. The MAC coefficients relating the sets of mode shapes, one or more of which occur in this interval, often do not show a clear closeness to either 1 or 0 for the coupled modes, even when the design variable changes between successive iterations are small.

An analytical logic is developed to detect the occurrence of the coupling condition and to identify the coupled modes. The program then temporarily suspends dealing with the untrackable modes until they are no longer detected as being coupled. To automate this procedure, an automatic recognition of the coupled modes is essential. The following presents a mathematical justification for the suggested method.

At the iteration k, $[\Phi^k]_J$ is determined using Eq. (3), and the following expression can be written:

$$[\Phi^{k-1}]_J = [\Phi^k]_J + [\Delta\Phi^k]_J \tag{4}$$

and

$$[\Delta\Phi^k]_J = [\Phi^k]_I [A^k]_{JI}^T \tag{5}$$

When modes are all uncoupled, elements of $[A^k]_{JI}$ are small.

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Table 1 Black Hawk correlation results using original PAREDYM

Iteration	Mode 1	2	3	4	5	6
			Frequencies f_i, cps			
0	5.336	6.361	9.429	11.011	12.348	12.746
1	5.354	6.471	9.699	11.376	12.799	13.117
2	5.474	6.602	10.005	11.721	13.141	13.186
3	5.636	6.817	10.382	12.200	13.636	13.878
4	5.581	6.817	10.559	12.449	13.465	13.788
Test	5.581	6.819	10.588	12.555	14.054	13.808
			Frequency errors $\langle \Delta f_i \rangle$			
0	0.0438	0.0672	0.1095	0.1230	0.1214	0.0770
1	0.0405	0.0510	0.0840	0.0939	0.0892	0.0501
2	0.0191	0.0319	0.0551	0.0664	0.0650	0.0451
3	0.0100	0.0003	0.0194	0.0283	0.0297	0.0051
4	0.0015	0.0004	0.0027	0.0084	0.0419	0.0014
			Mode shape errors $\langle \Delta \phi_i \rangle$			
0	0.1442	0.0469	0.0368	0.3899	0.1435	0.1013
1	0.1430	0.0464	0.0350	0.2645	0.1436	0.1006
2	0.1413	0.0465	0.0322	0.2157	1.3037	0.8933
3	0.1414	0.0476	0.0312	0.2184	0.1561	0.1076
4	0.1399	0.0486	0.0296	0.1840	2.1478	57.820

Table 2 Black Hawk correlation results using modified PAREDYM

Iteration	Mode 1	2	3	4	5	6
			Frequencies f_i, cps			
0	5.336	6.361	9.429	11.011	12.348	12.746
1	5.354	6.471	9.699	11.376	12.799	13.117
2	5.474	6.602	10.005	11.721	13.186	13.141
3	5.588	6.772	10.337	12.181	13.662	13.676
4	5.598	6.828	10.565	12.495	13.971	13.762
Test	5.581	6.819	10.588	12.555	14.054	13.808
			Frequency errors $\langle \Delta f_i \rangle$			
0	0.0438	0.0672	0.1095	0.1230	0.1214	0.0770
1	0.0405	0.0510	0.0840	0.0939	0.0892	0.0501
2	0.0191	0.0319	0.0551	0.0664	0.0617	0.0484
3	0.0013	0.0069	0.0237	0.0298	0.0268	0.0106
4	0.0031	0.0012	0.0022	0.0048	0.0059	0.0034
			Mode shape errors $\langle \Delta \Phi_i \rangle$			
0	0.1442	0.0469	0.0368	0.3899	0.1435	0.1013
1	0.1430	0.0464	0.0350	0.2645	0.1436	0.1006
2	0.1413	0.0465	0.0322	0.2157	0.1558	0.1231
3	0.1406	0.0470	0.0306	0.2134	0.1655	0.1174
4	0.1397	0.0482	0.0295	0.1885	0.1690	0.1340

Now, consider

$$[\Phi^{k-1}]_J^T[M^k][\Phi^k]_I = [\Phi^k]_J^T[M^k][\Phi^k]_I + [\Delta\Phi^k]_J^T[M^k][\Phi^k]_I$$
$$= [\Phi^k]_J^T[M^k][\Phi^k]_I + [A^k]_{JI}[\Phi^k]_I^T[M^k][\Phi^k]_I$$
$$= [J^k]_{JI} + [A^k]_{JI} \tag{6}$$

The matrix $[J^k]_{JI}$ contains 0 and 1.

Now, let

$$[H^k]_{JI} = [\Phi^{K-1}]_J^T[M^k][\Phi^k]_I \tag{7}$$

Then the inequality equation shown next should hold for all the corresponding elements of the matrices.

$$[|H^k|]_{JI} \le [|J^k|]_{JI} + [|A^k|]_{JI} \tag{8}$$

where $[|\bullet|]$ represents the matrix of the absolute values of its original elements. Therefore, we have

$$[G^k] = [|H^k|]_{JI} - [|J^k|]_{JI} \le [|A^k|]_{JI} \tag{9}$$

It is expected that the absolute values of all elements of $[G^k]$ are relatively small for noncoupling cases. Any elements of $[G^k]$ having relatively large absolute values are indications of possible coupled modes.

A partitioning vector can be formed by scanning the zero and nonzero elements of $[G^k]$. This partitioning vector can be used to partition out the untrackable modes from $[\Phi^k]_J$, the targeted modal matrix. That is,

$$[\Phi^k]_J = \left[\Phi_0^k \vdots \Phi_1^k\right] \tag{10}$$

where $[\Phi_0^k]$ and $[\Phi_1^k]$ are the untrackable modes and the active modes, respectively, at the iteration k.

The idea is to temporarily leave out the untrackable modes $[\Phi_0^k]$ and to target the reduced set of modes $[\Phi_1^k]$, for the iteration k. To resume tracking all of the targeted modes in the subsequent iterations, we need to restore a complete set of interested modes which are most updated and trackable. To do this we replace $[\Phi_0^k]$ with its corresponding set from the previous iteration (assuming those are trackable), i.e.,

$$\left[\tilde{\Phi}^k\right]_J = \left[\Phi_0^{k-1} \vdots \Phi_1^k\right] \tag{11}$$

Finally, we store $[\tilde{\Phi}^k]_J$ in place of $[\Phi^k]_J$ into the database and resume the normal iteration process.

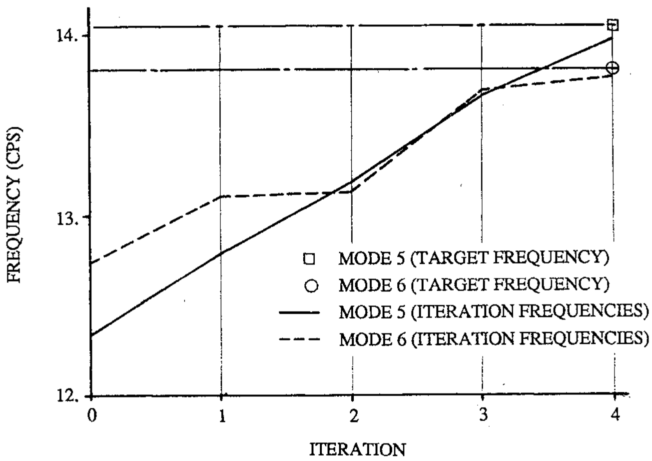

Fig. 1 Iteration history showing mode crossing phenomenon.

Demonstration Problem

This solution to the mode crossing problem has been implemented into a system identification program PAREDYM[2] for correlating MSC/NASTRAN modal analysis results with dynamic test results.[3] A practical example involving a large finite element (FE) model and real test results, is illustrated to demonstrate the effectiveness of the method.

A full-scale Sikorsky Black Hawk helicopter was used as the test structure for model correlation, and its actual natural frequencies and mode shapes were obtained through dynamic shake testing. A finite number of design variables, 33 in this case, were selected as variable parameters for the adjustment of the FE model.[4]

For each mode i, a measure of the eigenvalue error is denoted as

$$\langle \Delta f_i \rangle = \frac{f_i^e - f_i^a}{f_i^e} \tag{12}$$

where f_i^e represents the test frequency for the ith mode and f_i^a is the corresponding analysis value. The mode shape error for the ith mode is defined analogously.

For comparison purposes, the Black Hawk problem was rerun using both the original and the modified PAREDYM for four iterations, and the results are presented in Tables 1 and 2, respectively. In Table 1, it should be noted that the well-behaved steadily decreasing frequency errors shown here originally misled investigators into believing that these results were correct. The erratically deteriorating mode shape errors for modes 5 and 6, however, indicate that something is wrong. The modified PAREDYM, on the other hand, is able to recognize that three crossings have occurred and is able to correctly track the two modes through them (Fig. 1). Table 2 shows the near-continuous mode shape errors which result when the correct tracking is followed. That these errors do not now decrease in value, with successive iterations, is due to the fact that the mode shapes themselves are not being targeted here.

Conclusions

A simple and practical method has been introduced to handle the mode crossing condition, a common problem with iterative redesign procedures to modify structural modal parameters. The method has been applied successfully to modal correlation problems involving large FE models and real test data.

Based on the results of this study, the mode crossing condition can be detected using the modal assurance criterion, which compares the mode shapes of two consecutive iterations, and this MAC information can be used for a partition operation to correct the modal orders. The simplicity of this method permits it to be used in any iterative modal redesign procedure as a standard way of automatically preserving the correct modal orders with a minimum effort.

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